

SUPPLEMENT TO CHAPTER

4

Reliability

SUPPLEMENT OUTLINE

Introduction, 2

Finding Probability of Functioning
When Activated, 2

Finding Probability of Functioning
for a Given Length of Time, 4

Key Terms, 10

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LEARNING OBJECTIVES

*After completing this supplement,
you should be able to:*

- LO1** Define reliability and state two ways of using it.
- LO2** Find probability of functioning when activated, and explain the purpose of redundancy in a system.
- LO3** Find probability of functioning for a given length of time, and define failure rate per hour, mean time to failure, and availability.

reliability The ability of a product or part to perform its intended function under a prescribed set of conditions.

LO 1 INTRODUCTION

Reliability is a measure of the ability of a product or part to perform its intended function under a prescribed set of conditions. In effect, reliability is a *probability*.

Suppose that an item has a reliability of .90. This means that it has a 90 percent probability of functioning as intended. The probability that it will fail, i.e., its failure rate, is $1 - .90 = .10$, or 10 percent. Hence, it is expected that, on the average, 1 out of every 10 such items will fail or, equivalently, that the item will fail, on average, once in every 10 trials. Similarly, a reliability of .985 implies 15 failures per 1,000 parts or trials.

Reliability of a product or part is used in two ways.

1. Reliability when activated
2. Reliability for a given length of time

The first of these focuses on *one point in time* and is often used when a product or part must operate for one time, such as a missile or an air bag in a car. The second of these focuses on the *length of service*, such as most other products e.g., a car. The distinction will become more apparent as each of these approaches is described in more detail.

Reliability is an important dimension of product quality. Reliability management involves establishing, achieving, and maintaining reliability objectives for products, e.g., the expected life of a particular make of light bulb may be specified to be 5,000 hours. Achieving reliability usually falls on the shoulder of reliability engineers who use a variety of techniques to build reliability into products (e.g., by using reliable key components), test their performance, and estimate their reliability. If the reliability is inadequate, the types of failure and their effect on the product should be determined, their root cause(s) identified, and potential failure prevented.

We will mainly focus on reliability measurement, which involves statistics and probability theory. The average reliability of a part is measured by testing several units over time until some or all fail. However, this time may be very long (several years). To accelerate this, the items are stressed by using extreme environmental conditions such as high temperature, temperature cycles (e.g., hot-cold), high humidity, high vibration, high voltage, surges in power, etc. The resulting life estimate is then adjusted appropriately. Reliability of a product is determined from the reliability of its parts.

LO 2 FINDING PROBABILITY OF FUNCTIONING WHEN ACTIVATED

The probability that a part or product will operate as planned is an important concept in product design. Determining that probability when the product consists of a number of *independent* components requires the use of rules of probability for independent events. Independent events have no relation to the occurrence or nonoccurrence of each other. What follows are examples illustrating the use of two probability rules to determine whether a given product will operate successfully. Let P_i = probability that event i occurs, $i = 1, 2, 3, \dots$

Rule 1. If two or more events are independent and “success” is defined as the occurrence of all of the events, then the probability of success P_s is equal to the product of the probabilities of the events occurring, i.e., $P_s = P_1 \times P_2 \times \dots$

Example. Suppose a room has two lamps, but to have adequate light both lamps must work (success) when turned on. Here the product is the lighting system that has two component lamps. One lamp has a probability of working of .90, and the other has a probability of working of .80. The probability that both will work is $.90 \times .80 = .72$.

This lighting system can be represented by the following diagram where the two components are connected in series:



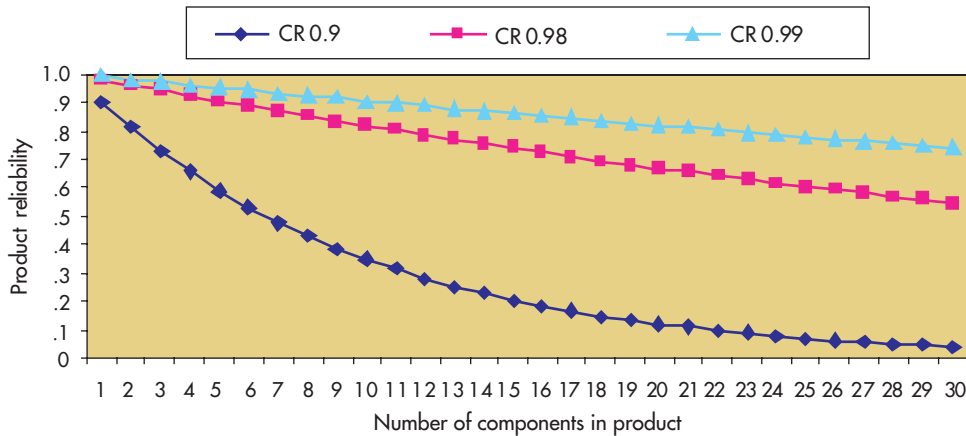


Figure 4S-1

Relating product and component reliabilities.

Even though the individual components of a series system (product) might have high reliability, the series system (product) as a whole can have considerably less reliability because all its components must function (i.e., the system is dependent on each of its components). As the number of components in a series system (product) increases, the system (product) reliability decreases. For example, a series system (product) that has eight components, each with a reliability of .99, has a reliability of only $.99^8 = .923$. See Figure 4S-1 for plots of product reliability as a function of number of its components for selected component reliability, CR.

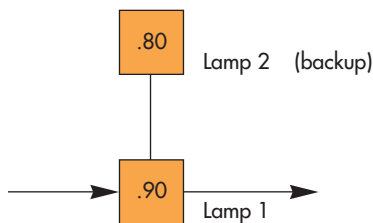
Many products have a large number of component parts that must all operate, and some way to increase overall reliability is needed. One approach is to overdesign, i.e., enhance the design to avoid a particular type of failure. For example, use a more durable and higher quality (but more expensive) material in a product. Another is design simplification, i.e., reduce the number of components in the product. The third approach is to use **redundancy** in the design. This involves providing backup components.

Rule 2. If two or more events are independent and “success” is defined as occurrence of *at least one* of the events, then the probability of success P_s is equal to $1 - \text{probability that none of the events will occur}$, i.e., $1 - (1 - P_1)(1 - P_2)(1 - P_3) \dots$. Simplifying, $P_s = P_1 + (1 - P_1)P_2 + (1 - P_1)(1 - P_2)P_3 + \dots$

Example. There are two lamps in a room. When turned on, one has probability of working of .90 and the other has probability of working of .80. Only a single lamp is needed to light the room for success (note that the threshold for success is different in this example). Then, probability of success $P_s = 1 - (1 - .90)(1 - .80) = .98$.

Conceptually, we can think of this system as a lamp with a *backup*. If the first lamp fails to light when turned on, the backup lamp is turned on. The probability of success P_s is probability that the first lamp operates plus probability that the first lamp fails and the backup lamp operates, i.e., $.90 + (1 - .90) \times .80 = .98$.

This backup system can be represented by the following diagram.



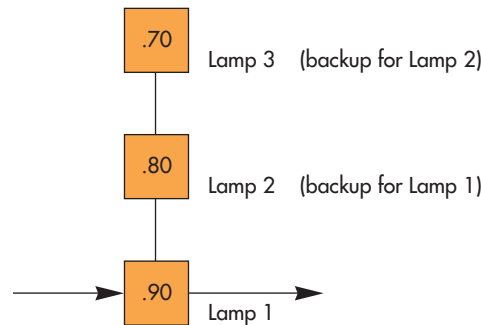
Example. Three lamps have probabilities of .90, .80, and .70 of lighting when turned on. Only one lighted lamp is needed for success. Then, probability of success $P_s = 1 - (1 - .90)(1 - .80)(1 - .70) = .994$.

redundancy The use of backup components to increase reliability.

Conceptually, we can think of this system as a lamp with a backup which in turn has a backup. If the first lamp fails to light when turned on, the second lamp is turned on. If the second lamp also fails to light when turned on, the third lamp is turned on. The probability of success P_s is probability that the first lamp operates plus probability that the first lamp fails and the second lamp operates plus probability that the first and second lamps fail and the third lamp operates, i.e.:

$$[\#1 \text{ operates}] + [\#1 \text{ fails and } \#2 \text{ operates}] + [\#1 \text{ fails and } \#2 \text{ fails and } \#3 \text{ operates}] \\ .90 + (1 - .90) \times .80 + [(1 - .90) \times (1 - .80) \times .70] = .994$$

This double backup system can be represented by the following diagram:



In general, a product (system) may be composed of some parallel components and some series components. The product's reliability is calculated in two stages: (a) first calculate the reliability of the parallel component(s) and then (b) use these to calculate the reliability of the resulting series system.

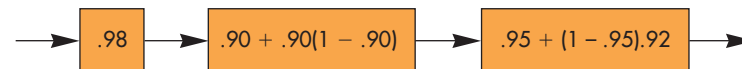
Example S-1

Determine the reliability of the system shown below.



Solution

(a) The system can be reduced to a series of three components:



(b) The system reliability is, then, the product of these component reliabilities:
 $.98 \times .99 \times .996 = .966$



FINDING PROBABILITY OF FUNCTIONING FOR A GIVEN LENGTH OF TIME

The second way of looking at reliability considers a use factor, usually the time dimension: probabilities are determined relative to a specified length of time. This approach is most common. Product warranties, e.g., one year free repair, should be based on this definition of reliability.

In this case, *failure rate per hour* is defined as the number of failures divided by total operating hours.

Example S-2

Two hundred units of a particular component were subjected to accelerated life testing equivalent to 2,500 hours of normal use. One unit failed after 1,000 hours and another after 2,000 hours. All other units were still working at the conclusion of the test.

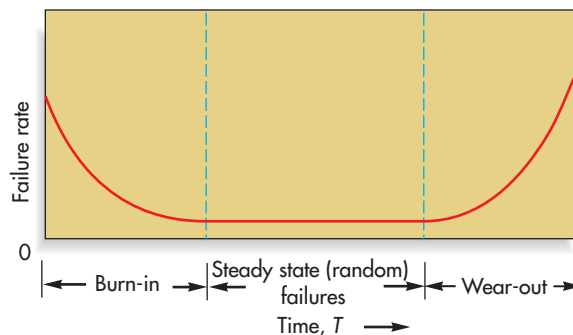
The failure rate per hour = $2/[198(2,500) + 1,000 + 2,000] = 0.000004016$ per hour

Note that this formula assumes constant failure rate over time.

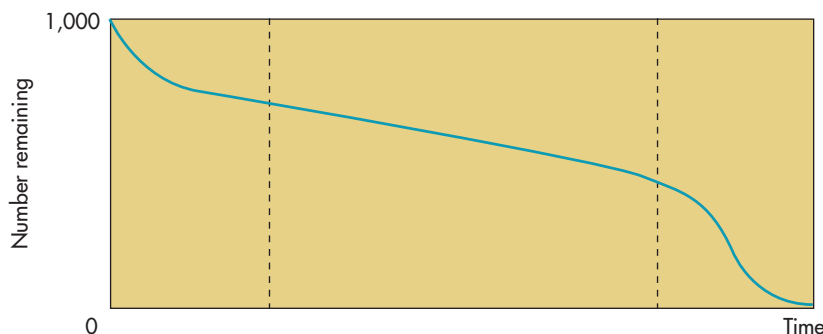
If failure rate is constant over time, *time-quantity transposition* is applicable, i.e., one can reduce the test time but use more test items. For example, in the above example, instead of 200 items tested for 2,500 hours, the reliability engineer could have used 400 items tested for 1,250 hours.

A typical profile of failure rate over time is shown in Figure 4S-2. Because of its shape, it is referred to as the *bathtub* curve. Usually, a number of products or parts fail shortly after they are put into service, because they are defective to begin with. Examples include electronics components such as capacitors. The rate of failure decreases rapidly as the defective items are weeded out. During the second phase, random failures occur. In many cases, this phase covers a relatively long period of time (several years). In the third phase, failures occur because the items are worn out, and the failure rate increases.

The following example illustrates the bathtub curve. In a reliability testing study, 1,000 light bulbs were lighted until they failed. Each failure time was recorded. The number of light bulbs remaining (survivors) over time can be seen in Figure 4S-3. Note that initially and close to the end, there are sharp drops in the number of light bulbs remaining (survivors), reflecting the burn-in and wear-out phases, respectively.

**Figure 4S-2**

Failure rate is generally a function of time and follows the bathtub curve.

**Figure 4S-3**

Number of light bulbs remaining over time.

mean time to failure (MTTF)

The average length of time before failure of a product or component.

mean time between failures (MTBF)

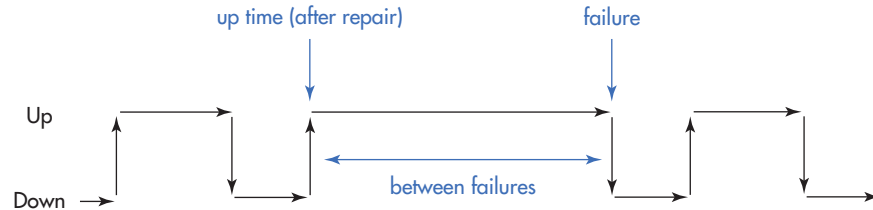
The average time from the up time after the repair following a failure to the next failure.

The inverse of failure rate per hour is **mean time to failure (MTTF)**, the average length of time (in hours) before failure. For data in Example S2,

$$\text{MTTF} = 1/\text{Failure rate per hour} = 1/0.000004016 = 249,000 \text{ hours}$$

Note that this formula assumes that failure rate is constant.

For repairable items, a similar term, **mean time between failures (MTBF)**, is usually used. MTBF is the average time from the up time after the repair following a failure to the next failure.



The time to failure of a non-repairable item during the steady state phase can often be modelled by the Exponential distribution with an average equal to the MTTF (see Figure 4S-4). Similar results hold for repairable items. The probability that the item put into service at time 0 will fail *before* some specified time, T , is equal to the area under the curve between 0 and T . Reliability of the item is the probability that it will last *at least until* time T ; therefore, reliability is equal to the area under the curve *beyond* T . (Note that the total area under the curve is 100 percent.) Observe that as the specified length of service increases, the area under the curve to the right of that point (i.e., the reliability of the item) decreases.

The Exponential distribution is completely described using a single parameter, its average, in this case the mean time to failure (or between failures). Using the symbol T to represent length of service, the reliability or probability that failure will not occur before time T (i.e., the area in the right tail) is easily determined by:

$$\text{Reliability} = P(\text{no failure before } T) = e^{-T/\text{MTTF}}$$

where

$$e \approx 2.7183$$

T = Length of service before failure

MTTF = Mean time to failure

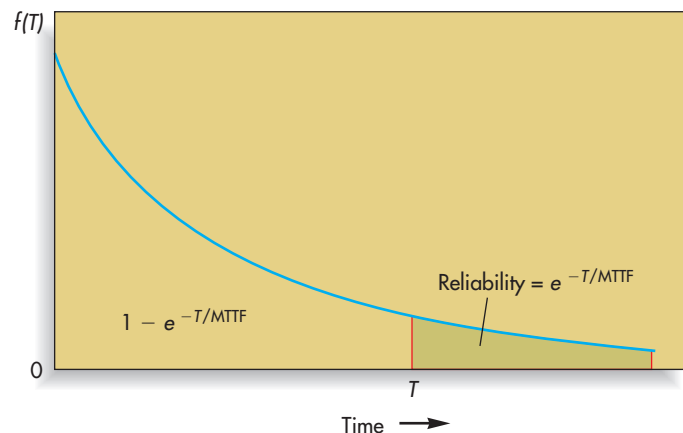
The probability that failure will occur before time T is 1 minus reliability:

$$P(\text{failure before } T) = 1 - e^{-T/\text{MTTF}}$$

Selected values of $e^{-T/\text{MTTF}}$ (i.e., reliability), given values for T/MTTF are listed in Table 4S-1.

Figure 4S-4

An Exponential distribution.



$T/MTTF$	$e^{-T/MTTF}$	$T/MTTF$	$e^{-T/MTTF}$	$T/MTTF$	$e^{-T/MTTF}$
.10	.9048	2.60	.0743	5.10	.0061
.20	.8187	2.70	.0672	5.20	.0055
.30	.7408	2.80	.0608	5.30	.0050
.40	.6703	2.90	.0550	5.40	.0045
.50	.6065	3.00	.0498	5.50	.0041
.60	.5488	3.10	.0450	5.60	.0037
.70	.4966	3.20	.0408	5.70	.0033
.80	.4493	3.30	.0369	5.80	.0030
.90	.4066	3.40	.0334	5.90	.0027
1.00	.3679	3.50	.0302	6.00	.0025
1.10	.3329	3.60	.0273	6.10	.0022
1.20	.3012	3.70	.0247	6.20	.0020
1.30	.2725	3.80	.0224	6.30	.0018
1.50	.2231	4.00	.0183	6.50	.0015
1.60	.2019	4.10	.0166	6.60	.0014
1.70	.1827	4.20	.0150	6.70	.0012
1.80	.1653	4.30	.0136	6.80	.0011
1.90	.1496	4.40	.0123	6.90	.0010
2.00	.1353	4.50	.0111	7.00	.0009
2.10	.1255	4.60	.0101		
2.20	.1108	4.70	.0091		
2.30	.1003	4.80	.0082		
2.40	.0907	4.90	.0074		
2.50	.0821	5.00	.0067		

Table 4S-1*Values of reliability = $e^{-T/MTTF}$*

By means of extensive testing and data collection, a manufacturer has determined that a particular model of its vacuum cleaners has an expected life that is Exponential with a mean of four years and insignificant burn-in phase. Find the probability that one of these vacuum cleaners will have a life that ends:

- After the initial four years of service.
- Before four years of service are completed.
- Not before six years of service.

MTTF = 4 years

- $T = 4$ years:

$$T/MTTF = \frac{4 \text{ years}}{4 \text{ years}} = 1.00$$

From Table 4S-1, $e^{-1.00} = .3679$.

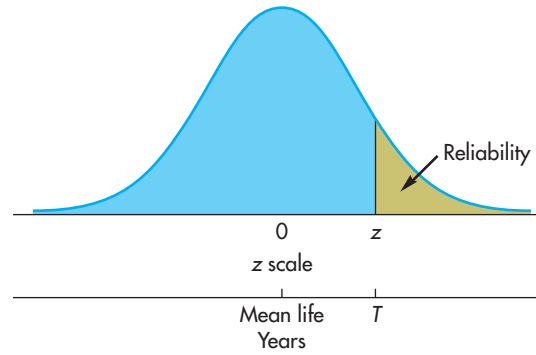
- The probability of failure before $T = 4$ years is $1 - e^{-1.00}$, or $1 - .3679 = .6321$.

- $T = 6$ years:

$$T/MTTF = \frac{6 \text{ years}}{4 \text{ years}} = 1.50$$

From Table 4S-1, $e^{-1.50} = .2231$.

Example S-3**Solution**

Figure 4S-5*A Normal curve*

Mechanical items such as ball bearings, valves, and springs tend to have insignificant burn-in and steady-state phases, and start to wear out right away. Item failure due to wear-out can sometimes be modelled by a Normal distribution. Obtaining Normal probabilities involves the use of the standard Normal table (see Appendix B, Table B). The table provides areas under a Normal curve up to a specified point z , where z is a *standardized* value calculated using the formula:

$$z = \frac{T - \text{Mean wear-out time}}{\text{Standard deviation of wear-out time}}$$

This area is the probability that service life will not exceed some value T . To find the reliability, subtract this probability from 1. See Figure 4S-5.

To obtain the value of T that will provide a given probability, work in reverse, i.e., locate the nearest probability in Appendix B, Table B, and pick up the associated z value. Then, insert the z value in the above formula and solve for T .

Example S-4

The mean life of a certain ball bearing can be modelled using a Normal distribution with a mean of six years and a standard deviation of one year. Determine each of the following:

- The probability that a ball bearing will fail *before* seven years of service.
- The probability that a ball bearing will fail *after* seven years of service (i.e., find its reliability).
- The service life that will provide a failure probability of 10 percent.

Solution

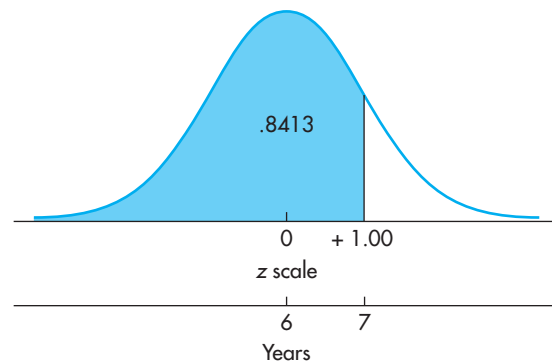
Wear-out mean = 6 years

Wear-out standard deviation = 1 year

Wear-out is Normally distributed

- Calculate z using the above formula:

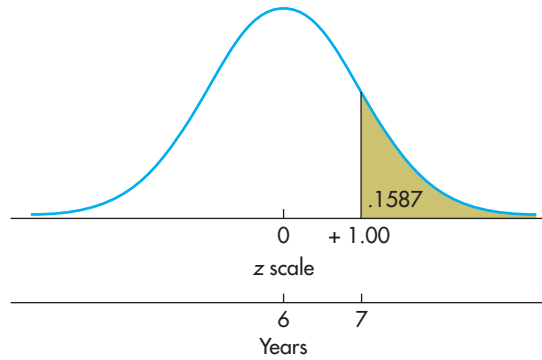
$$z = \frac{7 - 6}{1} = +1.00$$



Use the calculated z to obtain the required probability from Appendix B, Table B. Thus, $P(T < 7) = P(z < 1) = .8413$ (see the graph on the previous page).

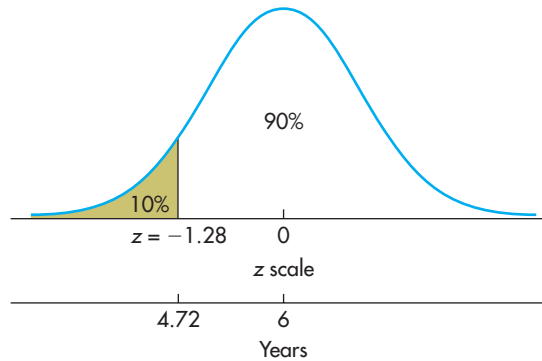
- b. Subtract the probability determined in part a from 1.00 (see the graph below).

$$1.00 - .8413 = .1587$$



- c. Use the standard Normal table in reverse, i.e., find the value of z that corresponds to an area under the curve (starting from the left side) of .10. Thus, $z = -1.28$ from Appendix B, Table B. Now, insert this in the z formula above:

$$z = 1.28 = \frac{T - 6}{1}$$



Solving for T , we find $T = 4.72$ years (see the graph above).

A more general distribution than Exponential is the Weibull distribution (see e.g., http://en.wikipedia.org/wiki/Weibull_distribution).

The probability density function of a Weibull random variable x is

$$f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda} \right)^{k-1} e^{-(x/\lambda)^k} & x \geq 0, \\ 0 & x < 0, \end{cases}$$

where $k > 0$ is the *shape parameter*, $\lambda > 0$ is the *scale parameter* of the distribution, and x represents time t . If $k = 1$ the failure rate is constant over time, and Weibull is identical to the Exponential distribution. If $k < 1$ failure rate decreases over time, and if $k > 1$ failure rate increases over time. Because of its flexibility, Weibull distribution is commonly used to model the time to failure during the burn-in ($k < 1$) and wear-out ($k > 1$) phases.

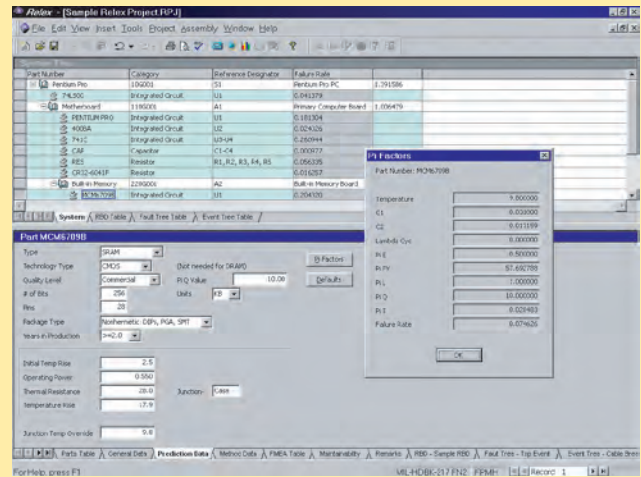
Please note that the probability rules of Section 2 above for series and parallel systems can also be used to determine reliability of a system for a given length of time based on the reliability of its components over the same length of time.



OM in ACTION

Windchill Quality Solutions (Formerly Relux Software)¹

Windchill Quality Solutions' reliability software assists in designing products and parts with reduced chance of failure, collecting reliability data, estimating failure rate and mean time between failures, finding the cause of failure, and many other reliability activities. To the right is a screenshot of the bill of material of the processing unit of a computer, with component failure rate (per million hours) provided from the library of the software, based on quality and expected operating conditions such as temperature.



Availability

availability The fraction of time a piece of equipment or a repairable product is expected to be available for operation.

mean time to repair The average length of time to repair a failed item.

For repairable items, the measure of importance to customers, and hence to designers, is **availability**. It measures the fraction of time a piece of equipment or a repairable product is expected to be available for operation (as opposed to being down for repair). Availability can range from zero (never available) to 1.00 (always available). Companies that can offer equipment with high availability have a competitive advantage over companies that offer equipment with lower availability. Availability is a function of both the mean time between failures and the **mean time to repair** (the average length of time to repair a failed item). We assume that there is little delay before a failed item begins to be repaired. The availability factor can be calculated using the following formula:

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

where

MTBF = Mean time between failures

MTTR = Mean time to repair

Example S-5

A copier is expected to operate for 200 hours after repair, and the mean repair time is expected to be two hours. Determine the availability of the copier.

MTBF = 200 hours, and MTTR = 2 hours

$$\text{Availability} = \text{MTBF} / (\text{MTBF} + \text{MTTR}) = 200 / (200 + 2) = .99$$

To increase availability, designers increase MTBF but also decrease MTTR. Laser printers, for example, are designed with print cartridges that can be easily replaced, thus requiring a small MTTR.

Key Terms

availability, 10

mean time between failures (MTBF), 6

mean time to failure (MTTF), 6

mean time to repair (MTTR), 10

redundancy, 3

reliability, 2

¹<http://www.ptc.com/products/windchill/quality>. Image courtesy PTC-Windchill Quality Solutions (formerly Relux Software Corporation).

A product designer must decide if a redundant component is cost-justified in a product. The product in question has a critical component with a probability of .98 of operating. Product failure would involve a cost of \$20,000. For a cost of \$100, a switch and backup component could be added that would automatically transfer the control to the backup component in the event of a failure. Should the backup component be added if its operating probability is also .98?

Because no probability is given for the switch, we will assume that its probability of operating when needed is 1.00. The expected cost of failure (i.e., without the backup) is \$20,000 $(1 - .98) = \$400$.

With the backup, the probability of *not* failing would be:
 $.98 + .02(.98) = .9996$

Hence, the probability of failure would be $1 - .9996 = .0004$. The expected cost of failure with the backup would be the added cost of the backup component plus the failure cost:

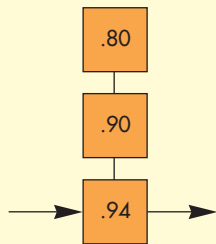
$$\$100 + \$20,000(.0004) = \$108$$

Because this (\$108) is less than the expected cost without the backup (\$400), adding the backup component is definitely cost-justified.

Due to the extreme cost of interrupting production, a manufacturer has two standby machines available in case a particular machine breaks down. The machine in use has a reliability of .94, and the backups have reliabilities of .90 and .80. In the event of a failure, a backup machine is brought into service. If this machine also fails, the other backup is used. Calculate the system reliability.

$$R_1 = .94, \quad R_2 = .90, \quad \text{and} \quad R_3 = .80$$

The system can be depicted in this way:



$$\begin{aligned} R_{\text{system}} &= R_1 + R_2(1 - R_1) + R_3(1 - R_2)(1 - R_1) \\ &= .94 + .90(1 - .94) + .80(1 - .90)(1 - .94) = .9988 \end{aligned}$$

A hospital has three *independent* fire alarm systems, with reliabilities of .95, .97, and .99. In the event of a fire, what is the probability that a warning would be given?

A warning would *not* be given if all three alarms failed. The probability that at least one alarm would operate is $1 - P(\text{none operate})$:

$$\begin{aligned} P(\text{none operate}) &= (1 - .95)(1 - .97)(1 - .99) = .000015 \\ P(\text{warning}) &= 1 - .000015 = .999985 \end{aligned}$$

Alternatively, $P(\text{warning}) = .95 + .97(1 - .95) + .99(1 - .95)(1 - .97) = .999985$

A weather satellite has expected life of 10 years from the time it is placed into Earth's orbit. Determine its reliability for each of the following lengths of service (assume that Exponential distribution is appropriate.)

- 5 years
- 12 years
- 20 years
- 30 years

Solved Problems

Problem 1

Solution

Problem 2

Solution

Problem 3

Solution

Problem 4

Solution

MTTF = 10 years

Calculate the ratio T/MTTF for $T = 5, 12, 20$, and 30 , and obtain the values of $e^{-T/\text{MTTF}}$ from Table 4S-1. These are the solutions (= reliabilities).

	T	MTTF	T/MTTF	$e^{-T/\text{MTTF}}$
a.	5	10	.50	.6065
b.	12	10	1.20	.3012
c.	20	10	2.00	.1353
d.	30	10	3.00	.0498

Problem 5

What is the probability that the satellite described in Solved Problem 4 will fail between 5 and 12 years after being placed into Earth's orbit?

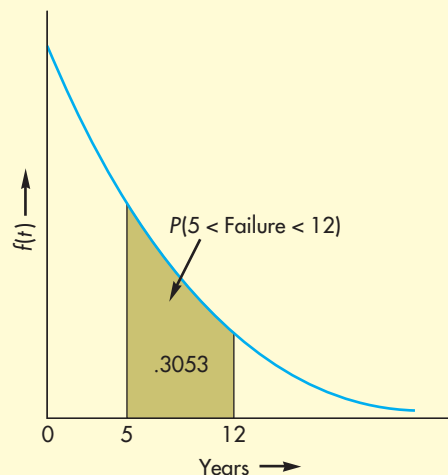
Solution

$$P(5 \text{ years} < \text{failure} < 12 \text{ years}) = P(\text{failure after 5 years}) \\ - P(\text{failure after 12 years})$$

Using the probabilities shown in the previous solution, we obtain:

$$P(\text{failure after 5 years}) = .6065 \\ - P(\text{failure after 12 years}) = .3012 \\ \hline .3053$$

See the following chart:

**Problem 6**

One line of specialty tires has a wear-out life that can be modelled using Normal distribution with a mean of 25,000 km and a standard deviation of 2,000 km. Determine each of the following:

- The percentage of tires that can be expected to wear out within $\pm 2,000$ km of the average (i.e., between 23,000 km and 27,000 km).
- The percentage of tires that can be expected to fail between 26,000 km and 29,000 km.
- For what tire life would you expect 4 percent of the tires to have worn out?

Solution

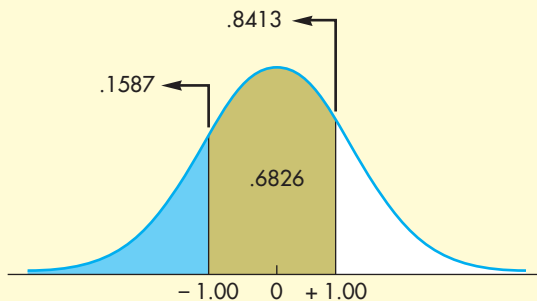
Note: Kilometres are analogous to time and are handled in exactly the same way.

- a. The phrase “within $\pm 2,000$ km of the average” translates to within one standard deviation of the mean because the standard deviation equals 2,000 km. Therefore, the range of z is $z = -1.00$ to $z = +1.00$, and the area under the curve between those points is found as the difference between $P(z < +1.00)$ and $P(z < -1.00)$, using values obtained from Appendix B, Table B.

$$P(z < +1.00) = .8413$$

$$-P(z < -1.00) = .1587$$

$P(-1.00 < z < +1.00) = .6826$, which means 68.26% of tires will wear out between 23,000 km and 27,000 km (see the following chart):



- b. Wear-out mean = 25,000 km

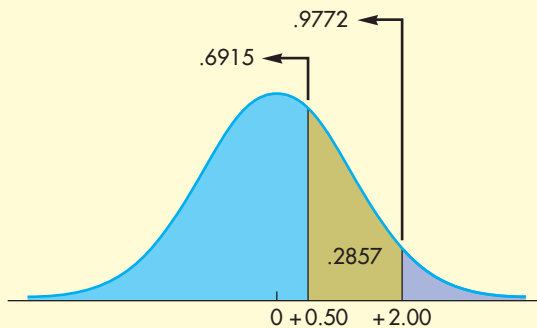
Wear-out standard deviation = 2,000 km

$$P(26,000 < \text{Wear-out} < 29,000) = P(z < z_{29,000}) - P(z < z_{26,000})$$

$$z_{29,000} = \frac{29,000 - 25,000}{2,000} = +2.00 \rightarrow .9772 \text{ (from Appendix B, Table B)}$$

$$z_{26,000} = \frac{26,000 - 25,000}{2,000} = +.50 \rightarrow .6915 \text{ (from Appendix B, Table B)}$$

The difference is $.9772 - .6915 = .2857$, which means 28.57 percent of tires will wear out between 26,000 km and 29,000 km (see the following chart).



- c. Use Appendix B, Table B to find z for 4 percent: $z = -1.75$
Find tire life using $\mu + z\sigma$: $25,000 - 1.75(2,000) = 21,500$ km.

Discussion and Review Questions

1. Define the term *reliability* and give an example. (LO1)
2. Explain why a product might have an overall reliability that is low even though its components have fairly high reliabilities. (LO2)
3. What is redundancy and how can it improve product reliability? (LO2)
4. How is failure rate per hour calculated? MTTF? Give an example of both. (LO3)
5. What is the significance of the bathtub curve in reliability? Give an example of an item with failure rate in each phase. (LO3)
6. How is reliability determined if the distribution of time to failure is Exponential? (LO3)
7. What is availability and how can it be increased? (LO3)

Internet Exercises

1. Visit either <http://www.smrpjobboard.com> or <http://www.sre.org/current/current.htm>, pick a reliability job announcement, and briefly summarize the duties involved. (LO1)
2. Read <http://asq.org/certification/reliability-engineer/bok.html>, and briefly summarize the knowledge and skills required of a Certified Reliability Engineer. (LO1)
3. Choose one of the following case studies and summarize it: (LO1)
 - a. <http://www.ptc.com/WCMS/files/114064/en/5018-RelexOpSim-Enphase-cs-en.pdf>
 - b. http://www.ptc.com/WCMS/files/114063/en/cs-4864-HR_Textron-Weibull.pdf
4. Visit <http://www.itl.nist.gov/div898/handbook/apr/section1/apr18.htm>, find the information about the Standby System and parallel system, and briefly explain how they differ. (LO3)
5. Visit <http://www.sqconline.com/resistor-mtbf-mil-hdbk-217-rev-f-notice-2>, find the failure rate and MTBF of a resistor, and identify the relationship between the two. (LO3)

Problems

1. Consider the following system: (LO2)



Determine the probability that the system will operate under each of these conditions:

- a. The system as shown.
 - b. Each system component has a backup with a reliability of .90 and a switch that is 100 percent reliable.
 - c. Each system component has a backup with .90 reliability and a switch that is 99 percent reliable.
2. A product is composed of four parts. In order for the product to function properly, each of the parts must function. Two of the parts each have .96 probability of functioning, and the other two each have probability of .99. What is the overall probability that the product will function properly? (LO2)
 3. A system consists of three identical components. In order for the system to perform as intended, all of the components must perform. Each component has the same probability of performance. If the system is to have .92 probability of performing, what probability of performance is needed by each of the individual components? (LO2)
 4. A product engineer has developed the following equation for the cost of a component: $C = (10P)^2$, where C is the cost in dollars and P is the probability that the component will operate as expected. The product is composed of two of these components, both of which must operate for the product to operate. The engineer can spend a total of \$173 for the two components. To the nearest two decimal places, what is the largest component reliability that can be achieved? (LO2)
 5. The guidance system of a ship is controlled by a computer that has three major modules. In order for the computer to function properly, all three modules must function. Two of the modules have reliability of .97, and the other has reliability of .99. (LO2)
 - a. What is the reliability of the computer?
 - b. A backup computer identical to the one being used can be installed to improve overall reliability. Assuming that the new computer can automatically function if the first computer fails, determine the resulting reliability.
 - c. If the backup computer must be activated by a switch in the event that the first computer fails, and the switch has a reliability of .98, what is the overall reliability of the system?

(Both the switch and the backup computer must function in order for the backup system to function.)

6. One of the industrial robots designed by a leading producer has four major components. Components' reliability are .98, .95, .94, and .90. All of the components must function in order for the robot to operate effectively. (LO2)
 - a. Calculate the reliability of the robot.
 - b. Designers want to improve the reliability of the robot by adding a backup component. Due to space limitations, only one backup can be added. The backup for any component will have the same reliability as the unit for which it is the backup. Which component should get the backup in order to achieve the highest reliability of the robot?
 - c. If one backup with a reliability of .92 can be added to any one of the main components, which component should get it to achieve the highest overall reliability?
7. A production line has three machines A, B, and C, with reliabilities of .99, .96, and .93, respectively. The machines are arranged so that if one breaks down, the others must shut down. Engineers are weighing two alternative designs for increasing the line's reliability. Plan 1 involves adding an identical backup line (i.e., a series backup), and plan 2 involves providing a backup for each machine (i.e., a parallel backup). In either case, three additional machines (A, B, and C) would be used with reliabilities equal to the original three. (LO2)
 - a. Which plan will provide higher reliability?
 - b. Explain why the two reliabilities are not the same.
 - c. What other factors might enter into the decision of which plan to adopt?
8. Refer to the previous problem. (LO2)
 - a. Assume that a single switch is used in plan 1 to transfer production to the backup line if the first line failed, and this switch is 98 percent reliable, while reliabilities of the machines remain the same. Recalculate the reliability of plan 1. Compare this reliability with the reliability of plan 1 calculated in solving the original problem. How much did reliability of plan 1 decrease as a result of a 98-percent-reliable switch?
 - b. Assume that three switches are used in plan 2 to transfer production to the backup machines if the original machines failed, and these switches are all 98 percent reliable, while reliabilities of the machines remain the same. Recalculate the reliability of plan 2. Compare the reliability of this plan with the reliability of plan 2 calculated in solving the original problem. How much did reliability of plan 2 decrease?
9. A Web server has five major components that must all function in order for it to operate as intended. Assuming that each component of the system has the same reliability, what is the reliability each one must have in order for the overall system to have a reliability of .98? (LO2)
10. Repeat Problem 9 under the condition that one of the components will have a backup with reliability equal to that of any one of the other components. (LO2)
11. Hoping to increase the chances of reaching a performance goal, the director of a research project has assigned the same task to three separate research teams. The director estimates that the team probabilities for successfully completing the task in the allotted time are .9, .8, and .7. Assuming that the teams work independently, what is the probability that the project will not be completed in time? (LO2)
12. An electronic chess game has a useful life that is Exponential with a mean of 30 months. Determine each of the following: (LO3)
 - a. The probability that any given unit will operate for at least (1) 39 months, (2) 48 months, (3) 60 months.
 - b. The probability that any given unit will fail sooner than (1) 33 months, (2) 15 months, (3) 6 months.
 - c. The length of service time after which the percentage of failed units will approximately equal (1) 50 percent, (2) 85 percent, (3) 95 percent, (4) 99 percent.
13. A manufacturer of programmable calculators is attempting to determine a reasonable warranty period for a model it will introduce shortly. The manager of product testing has indicated that the calculators have an expected life of 30 months. Assume product life can be described by Exponential distribution. (LO3)
 - a. If warranties are offered for the expected life of the calculators, what percentage of those sold would be expected to fail during the warranty period?

- b. What warranty period would result in a failure chance of approximately 10 percent?
- 14. A type of light bulb has a life that is Exponentially distributed with a mean of 5,000 hours. Determine the probability that one of these light bulbs will last: (LO3)
 - a. At least 6,000 hours.
 - b. No longer than 1,000 hours.
 - c. Between 1,000 hours and 6,000 hours.
- 15. According to its designers, a satellite will have an expected life of six years. Assume that Exponential distribution applies. Determine the probability that it will function for each of the following time periods: (LO3)
 - a. More than 9 years.
 - b. Less than 12 years.
 - c. More than 9 years but less than 12 years.
 - d. At least 21 years.
- 16. An office manager has received a report from a consultant on equipment replacement. The report indicates that the scanners have a service life that is Normally distributed with mean of 41 months and standard deviation of 4 months. On the basis of this information, determine the percentage of scanners that can be expected to fail in the following time periods: (LO3)
 - a. Before 38 months of service
 - b. Between 40 and 45 months of service
 - c. Within 2 months of the mean life
- 17. A copier manufacturer has determined that its major product has a service life that can be modelled by Normal distribution with mean of six years and standard deviation of half year. (LO3)
 - a. What probability can you assign to service lives of (1) at least five years? (2) at least six years? (3) at most seven and a half years?
 - b. If the manufacturer offers service warranty of four years on these copiers, what percentage can be expected to fail during the warranty period?
- 18. Refer to Problem 17. What warranty period would result in percentage failure of: (LO3)
 - a. 2 percent?
 - b. 5 percent?
- 19. Determine the availability for each of these cases: (LO3)
 - a. MTBF = 40 days, MTTR = 53 days
 - b. MTBF = 300 hours, MTTR = 6 hours
- 20. A machine can operate for an average of 50 days before it needs to be overhauled, a process that takes two days. Calculate the availability of this machine. (LO3)
- 21. A manager must decide between two machines. Machine A has an average operating time of 142 hours and an average repair time of 7 hours. Times for machine B are an average operating time of 65 hours and an average repair time of 2 hours. What is the availability of each machine? (LO3)
- 22. A designer estimates that she can (a) increase the average time to failure of a product by 5 percent at a cost of \$450, or (b) reduce the average repair time by 10 percent at a cost of \$200. Which option would be more cost-effective? Currently, the average time to failure is 100 hours and the average repair time is 4 hours. (LO3)
- 23. A battery's life is Normally distributed with mean of 4.7 years and standard deviation of .3 year. The batteries are warrantied to operate for a minimum of four years. If a battery fails within the warranty period, it will be replaced with a new battery at no charge. (LO3)
 - a. What percentage of batteries would you expect to fail before the warranty period expires?
 - b. The manager is toying with the idea of using the same battery with a different exterior, labelling it as a premium battery, and offering a 54-month warranty on it. What percentage of "premium" batteries would you expect to fail before the warranty period expires?
- 24. In practice, for a series system the failure rate is estimated by adding the failure rate of its components. For a system made of n identical components in series, each having a probability

of failure = P_f , probability of system failure is approximately $n(P_f)$ provided that P_f is sufficiently small. Choose a value of $n > 1$ and $P_f < .05$, and show the above result. (LO2)

25. The MTTF of the central processing unit (CPU) of a single board computer is estimated to be 150,000 hours. You can assume Exponential distribution for operating time of this component until failure. What is the probability that this component will operate without failure for: (LO3)
- 2.5 years?
 - 5 years?
 - 10 years?

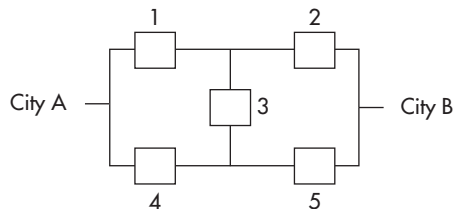
(Hint: Use a calculator, instead of Table 4S-1, to obtain more accurate probabilities.)

- *26. A study was performed to determine the reliability of components of personal computers (PCs) used by Rolls-Royce staff.² The operating lives of the components of 341 PCs were measured over a 22-month period. The study fit the Weibull distribution to the sample operating lives of each component and based on the estimated parameters k (shape) and λ (scale), calculated its average life or MTTF. (LO3)

$$f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda} \right)^{k-1} e^{-(x/\lambda)^k} & x \geq 0, \\ 0 & x < 0, \end{cases}$$

In the above formula, x represents time. The mean of the Weibull distribution is $\lambda \Gamma(1 + 1/k)$ where $\Gamma(y)$ can be computed in Excel using “=EXP(GAMMALN(y))”. While some components had a larger failure rate at the beginning of their life (e.g., hard disks with $k = .51$), others had almost constant failure rate over time (e.g., motherboards with $k = .99$). Mice had $k = .86$, $\lambda = 22,440$, and average life of 24,000 hours; keyboards had $k = .76$, $\lambda = 41,919$, and average life of 49,000 hours; hard disks had $k = .51$, $\lambda = 136,752$, and average life of 264,000 hours; and monitors had $k = .76$, $\lambda = 58,395$, and average life of 69,000 hours.

- For motherboards, $k = .99$ and $\lambda = 49,171$. Estimate the average life of a motherboard.
 - Suppose 90 motherboards (out of 341) failed during the study period (22 months), and their lives added to 540,000 hours. Estimate the average life of a motherboard (in hours) directly (i.e., not using Weibull distribution).
27. A car has four independent and identical tires. The reliability of a tire is .99. If any tire is flat, the car cannot be driven. Calculate the reliability of a car with respect to its tires. (LO2)
28. A computer has two independent and identical Central Processing Units (CPUs). The computer will operate if at least one CPU operates. The reliability of a CPU is .99. Calculate the reliability of the computer with respect to its CPUs. (LO2)
- *29. A communication network between two cities, A and B, consists of five independent and identical relay units forming a bridge network as shown below. For the network to work, a least one path between the two cities should work. If the reliability of each relay is .99, what is the reliability of the network R_N ? Hint: $R_N = 1 - \text{sum of probability that any minimal two or three relays that together cut off the network will fail simultaneously}$. Specifically, any of sets of relays 1&4, 2&5, 1&3&5, and 2&3&4 cut off the two cities. (LO2)



²M. Bradley and R. Dawson, “The Cost of Unreliability: A Case Study,” *Journal of Quality in Maintenance Engineering*, 4(3), 1998, pp. 212–218.

30. A plane has two independent and identical engines. At least one engine should operate for the plane not to crash. The reliability of an engine is .999. Calculate the reliability of the plane with respect to its engines. (LO2)
31. Suppose that the failure rate of a tire is .0001 per hour of driving (with time to failure being Exponential), and that the failure rates of the four tires are independent and identical. (LO3)
- What is the failure rate of the car with respect to its tires? *Hint:* It is the sum of the failure rate of the four tires because any tire's failure would result in car's failure.
 - Calculate the mean time to a tire failure for the car.
 - Calculate the reliability of the car with respect to its tires if a 25-hour journey is going to take place.

Hint: The distribution of time to failure for the car is also Exponential.

- *32. A computer has two independent and identical Central Processing Units (CPUs) that are both used when the computer is on. Suppose that the failure rate of a CPU is .0001 failures per hour (with time to failure being Exponential), and the computer will operate if at least one CPU operates. Calculate the mean time to CPU failure of the computer. *Hint:* It can be shown that

$$MTTF_p = \frac{1}{\lambda} \sum_{i=1}^m \frac{1}{i}$$

where λ is the failure rate of one CPU and m is the number of parallel CPUs (in this case $m = 2$). (LO3)

- *33. A standby system consists of two independent and identical units. When the first unit fails, the second unit (standby) kicks in. Note that unlike the parallel system of the previous problem, the standby component is brought into operation only when needed. A unit's failure rate is .0001 failures per hour (with time to failure being Exponential). Calculate the system mean time to failure (i.e., both units failing). *Hint:* It can be shown that

$$MTTF = \frac{m}{\lambda}$$

where λ is the failure rate of a unit and m is the number of units (in this case $m = 2$). (LO3)

- *34. The following data shows the time to failure of 100 units of a generator that were run until failure.³ (LO3)
- Compute the failure rate during each month as a proportion of numbers that survived until the beginning of that month. Draw these failure rates against months. Determine if the failure rate is decreasing, constant, or increasing.
 - If we assume that failure rate is constant, determine the average failure rate (per month) and MTTF (in months).

Month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No. failed	1	0	2	1	4	10	18	22	14	8	10	5	3	1	0	1

35. Five units of an electronic component were subjected to stress testing until they failed⁴. The observed failure times (in days) were 550, 680, 820, 910, and 1,110. Calculate the MTTF and the failure rate for this component. (LO3)
36. Twenty units of a component with constant failure rate were subjected to high-stress testing. After 25 hours, seven failed at times (in hours) 2.1, 8.3, 10.9, 15.2, 16.3, 20.5, 23.8.⁵ Calculate the MTTF and the failure rate for this component. (LO3)
- *37. The following data shows the number of failures of 1,000 units of an electronic component in time intervals of 100 hours.⁶ (LO3)
- Compute the failure rate during each time interval as a proportion of numbers that survived until the beginning of that interval. What can you conclude about the failure rates?
 - Calculate the MTTF for this component.

³R. D. Leitch, *Basic Reliability Engineering Analysis*, London: Butterworths, 1988, p. 21.

⁴R. D. Leitch, *Basic Reliability Engineering Analysis*, London: Butterworths, 1988, p. 33.

⁵R. D. Leitch, *Basic Reliability Engineering Analysis*, London: Butterworths, 1988, p. 34.

⁶D. J. Klinger et al., Editors, *AT&T Reliability Manual*, New York: Van Nostrand Reinhold, 1990, p. 7.

Time Interval	No. of Failures
0–100	95
100–200	86
200–300	78
300–400	70
400–500	64
500–600	58
600–700	52
700–800	47
800–900	42
900–1000	39

38. A piece equipment contains 300 integrated circuits, 25 amplifiers, 150 transistors, 500 resistors, and 450 capacitors.⁷ Failure of any component causes a system failure. Using the estimated failure (hazard) rates below, calculate the failure (hazard) rate of the equipment. (LO3)

Component	Hazard rate (failures per billion hour)
Integrated circuit	10
Amplifier	30
Transistor	20
Resistor	1
Capacitor	1



MINI-CASE

Engineer Tank⁸

Engineer tank is an armoured vehicle used on the battlefield to perform engineering tasks such as opening routes, digging, and bulldozing. During design, the mean time between

the need for unscheduled maintenance (other than loss of propulsion) was set to 420 hours. (The target for MTBF for loss of propulsion is 3,500 hours). During reliability testing for 70 engineer tanks, the need for unscheduled maintenance arose after the following number of hours: (LO3)

No.	Failed hr	No.	Failed hr	No.	Failed hr	No.	Failed hr	No.	Failed hr	No.	Failed hr	No.	Failed hr
1	9	11	52	21	105	31	178	41	232	51	284	61	416
2	18	12	64	22	114	32	194	42	232	52	291	62	425
3	22	13	68	23	122	33	195	43	239	53	292	63	441
4	23	14	75	24	126	34	198	44	241	54	294	64	451
5	27	15	88	25	126	35	202	45	244	55	315	65	454
6	33	16	89	26	130	36	210	46	247	56	323	66	456
7	36	17	91	27	151	37	215	47	247	57	325	67	557
8	42	18	94	28	153	38	216	48	252	58	350	68	560
9	46	19	95	29	165	39	223	49	273	59	360	69	947
10	50	20	99	30	168	40	223	50	275	60	378	70	997

- Make a histogram of these times. Does the distribution look like Exponential?
- Compute the mean. Is the target MTBF met?

⁷D. J. Klinger et al., Editors, *AT&T Reliability Manual*, New York: Van Nostrand Reinhold, 1990, p. 94.

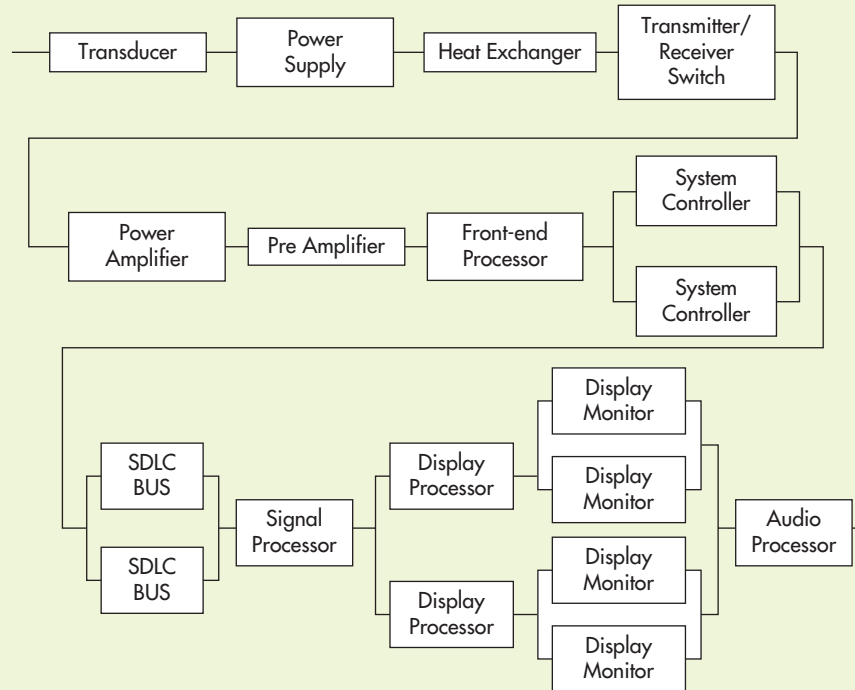
⁸U. D. Kumar et al., *Reliability and Six Sigma*, New York: Springer, 2006, p. 102.



MINI-CASE

Sonar System⁹

The components of an underwater sonar system are given in the block diagram below:



The failure rate of one unit of each component is estimated below using available data tables:

Subsystem	Failure rate $\lambda/10^6$ hours
Transducer	1,000
Power Supply	20
Heat Exchanger	20
Transmitter/Receiver Switch	6
Power Amplifier	120
Pre Amplifier	32
Front-end Processor	400
System Controller (parallel)	10
SDLC Bus (parallel)	15
Signal Processor	450
Display Processor (parallel)	150
Display Monitor (parallel)	50
Audio Processor	20

Calculate the reliability of the sonar system to work 100 hours failure-free. *Hint:* For 2 parallel components, the overall failure rate of the two is the failure rate of one divided by 1.5 (see Problem 32 above).

⁹U. D. Kumar et al., *Reliability and Six Sigma*, New York: Springer, 2006, pp. 139–141.